Unbinding the Deuteron

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Abstract

We consider a description of the deuteron based on meson exchange potentials. A key feature is the inclusion of the $I = S = 0$ two-pion intermediate state ('$\sigma(600)$') as a significant component of the inter-nucleon potential energy. In this approach, deuteron binding is seen to be predominantly a consequence of $\sigma(600)$ and $\omega(783)$ exchange, with a secondary role played by $\rho(770)$. We explore sensitivity of two-nucleon binding to changes in the potential and thereby obtain an anthropic constraint — that the deuteron unbinds for a modest decrease (about 6%) in the attractive $\sigma(600)$ potential.

I. INTRODUCTION

Imagine a sequence of worlds in which the light quark masses are continuously varied away from their physical values. It has been argued that heavy nuclei will disassociate for a 64% increase in the sum $m_u + m_d$ [1]. One can anticipate that before this happens, the deuteron will become unbound. This is because the deuteron binding energy is less than the average binding energy per nucleon. Here, we study this problem quantitatively and provide details about how the deuteron would respond to variations in its potential induced by changes in $m_u + m_d$ [2].

The deuteron is an example of a hadronic molecule. Such a two-hadron composite, bound by the strong interactions and whose mass lies slightly below the associated two-hadron threshold, should be describable with a nonrelativistic Schrödinger equation. We can picture the two hadrons as interacting via a meson exchange potential [3]. Hadronic, as opposed to quark, degrees of freedom should be appropriate for these situations. In this paper, we consider the S-wave radial Schrödinger equation,

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dr^2} + V(r) - E \right] u(r) = 0 \quad , \tag{1}$$

where M is the reduced mass of the two hadrons and $u(0) = u(\infty) = 0$.

In Section II, our focus will be on the form of the potential energy function $V(r)$, especially as regards the contributions from multi-pion exchanges. In Section III, we carry out a numerical study of how varying the potential energy affects deuteron binding. Our conclusions and plans for future study are presented in Section IV.

II. MULTI-PION EXCHANGE IN DEUTERON BINDING

Although the deuteron state, and more generally the two-nucleon potential, are long-studied areas of research, they continue to attract theoretical attention. There has been much recent activity about how the nuclear potential behaves in the chiral limit of zero light quark mass [4–6]. This work has motivated our interest in deuteron binding. By design our description is simple, *e.g.* we omit tensor interactions and thus our 'deuteron' has no D-wave component [7]. We comment further on this aspect of our work in the Conclusion.

Spatial potential energy functions for nonrelativistic calculations are obtained from particle exchange diagrams in quantum field theory. A nonrelativistic reduction of the exchange amplitude is followed by Fourier transformation from momentum to position space (see Eqs. (2)-(3) below). For example, photon exchange between an electron and a proton leads to a number of separate

effects (collectively the *Breit-Fermi* interaction [8]) starting with the long range Coulomb potential. In addition to generating long range potentials, such exchange amplitudes also generally give rise to short range or even local contributions. In this paper, we restrict our attention to only the long range part.

A. Two-pion and Three-pion Exchange Potentials

For convenience, we shall cast the potential energy in a dispersive form [9]. The strongest components of the two-nucleon potential arise from the isoscalar ($I = 0$) two-pion and three-pion exchange channels. A smaller isovector two-pion component must also be included.

For isoscalar (I=0) potentials, the two-pion (scalar) exchange gives rise to an attractive interaction while the three-pion (vector) interaction is repulsive. A momentum space version is given in terms of a corresponding spectral function $\rho_i^{(I=0)}(\mu)$,

$$V_i^{(I=0)}(q^2) = \frac{1}{\pi} \int_{\mu_i^2}^{\infty} d\mu^2 \, \frac{\rho_i^{(I=0)}(\mu)}{\mu^2 + q^2} \quad , \tag{2}$$

where $i = S, V$ and $\mu_S^2 = (2m_\pi)^2, \mu_V^2 = (3m_\pi)^2$. The spatial potential energie $V_i^{(I=0)}(r)$ corresponding to Eq. (2) is

$$V_i^{(I=0)}(r) = \frac{1}{4\pi^2 r} \int_{\mu_i^2}^{\infty} d\mu^2 \, e^{-\mu r} \rho_i^{(I=0)}(\mu) \quad . \tag{3}$$

The above unsubtracted dispersive forms are taken from Refs. [6, 10], where behavior at relatively low energies (< 1 GeV) is studied. The issue of high energy behavior and subtractions is considered in Ref. [11]. However, the choice of a subtraction scale at sufficiently high energy is not expected to alter the basic conclusions of Refs. [6, 10], which are anchored by the constraints of chiral perturbation theory [12].

For the isoscalar vector channel, it is a good approximation to take $\rho_V^{(I=0)}(\mu) \propto \delta(\mu - m_\omega)$, where $m_\omega \simeq 783$ MeV. This is also appropriate for the isovector two-pion exchange channel, where now $\rho_V^{(I=1)}(\mu) \propto \delta(\mu - m_\rho)$, with $m_\rho \simeq 770$ MeV. In these two cases, the association between an exchanged vector meson of mass m_V and a spatial Yukawa potential $V_V(r) \propto e^{-m_V r}$ is natural and obvious. The isoscalar two-pion exchange channel is more subtle.

B. The Two-pion Isoscalar ('$\sigma(600)$') Component of the Two-nucleon Potential

The nucleon-nucleon potential is known to have a strong attractive contribution of intermediate range. This arises from an exchange potential in the scalar, isoscalar channel, due to the exchange

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of a pair of pions in the $I = S = 0$ channel. There is now a description of this which, although not explicitly given in terms of quark interactions, does nonetheless respect the underlying QCD dynamics. The approach is (i) to use chiral perturbation theory to describe the low energy part of the momentum-space potential, and (ii) to invoke two-pion rescattering via the Omnes function at higher energies in order to include the constraint of unitarity.

The chiral component of $\rho_S^{(I=0)}$ turns out, as is generally the case in chiral perturbation theory, to depend on a small number of low energy constants whose numerical values are obtained phenomenologically. To the extent that these constants depend on any meson, it is actually $\rho(770)$ that contributes. The chiral form of $\rho_S^{(I=0)}$ grows with energy and at some point requires modification in order to respect unitarity. Such a modification can be accomplished [6] by introducing the Omnes function,

$$\Omega^{(I=0)}(\mu) = \exp\left[\frac{\mu^2}{\pi}\int \frac{ds}{s}\,\frac{\delta^{(I=0)}(s)}{s - \mu^2}\right] \quad, \tag{4}$$

where $\delta^{(I=0)}(s)$ is the S-wave two-pion phase shift in the $I = 0$ channel. [1] In this manner, the two ingredients of chiral perturbation theory and two-pion rescattering provide a sound theoretical basis for understanding the two-pion $S = I = 0$ exchange potential.

In the intermediate energy range $400 \leq E_{\pi\pi} \leq 700$ MeV, the $S = I = 0$ two-pion phase shift does not pass through 90º and thus (unlike $\omega(783)$ and $\rho(770)$) has no obvious association with a resonant state. The dispersive spatial potential of Eq. (3) would seem to require a broad spectral function in this case. However, it can be shown (see especially Fig. 3(b) of Ref. [6]) that *as a numerical recipe* the effect of nonresonant $S = I = 0$ two-pion scattering is reproduced by using $V_\sigma(r) \propto e^{-m_\sigma r}$ with $m_\sigma \simeq 600$ MeV. In our work, we adopt this approach and refer to it hereafter as the '$\sigma(600)$ exchange potential'. We stress that our use of '$\sigma(600)$' is simply a convenient shorthand and not meant to commit us to any specific model of this state [14].

III.   BINDING THE DEUTERON

Let us now consider the two-nucleon interaction in terms of four exchange potentials,

$$V^{(NN)} = V_\sigma^{(NN)} + V_\rho^{(NN)} + V_\omega^{(NN)} + V_\pi^{(NN)} \quad. \tag{5}$$

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[1] It is also possible to proceed to a higher order of the chiral expansion [13].

Each of these is taken to have the Yukawa form,

$$V^{(\mathrm{NN})}(r) = \sum_i \eta_i \frac{g_i^2}{4\pi} \frac{e^{-m_i r}}{r} \quad , \tag{6}$$

where $\eta_i = +1$ for repulsion and $\eta_i = -1$ for attraction. All that remains is to fix the strength of each potential.

For detailed discussions of potential energy fits to nuclear binding, see Refs. [10, 15]. We will assume the following values for the $\sigma(600)$ and $\omega(783)$ couplings [16],

$$\frac{g_{\omega\mathrm{NN}}^2}{4\pi} \simeq 16.6 \qquad \text{and} \qquad \frac{g_{\sigma\mathrm{NN}}^2}{4\pi} \simeq 10.3 \rightarrow 12.9 \quad . \tag{7}$$

Our strategy will be to fit $g_{\sigma\mathrm{NN}}^2/4\pi$ to the experimental deuteron binding energy [17]

$$\mathrm{BE}_{\mathrm{deut}} = 2.22457 \text{ MeV} \quad \text{(with negligible error)} \tag{8}$$

and see how it compares to the range in Eq. (7). We use this approach because a precise fit will allow us to study the effect on $\mathrm{BE}_{\mathrm{deut}}$ of modifying the potential energy function.

The pion potential energy is obtained by starting from the field theory interaction with pseudoscalar coupling

$$\mathcal{L}_{\pi\mathrm{NN}} = i f_{\pi\mathrm{NN}} \bar{N} \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} N \tag{9}$$

and reducing the pion exchange graph to its nonrelativistic limit. Upon keeping only the Yukawa dependence of Eq. (6), one obtains the isospin-dependent, spin-dependent potential energy

$$V_\pi^{(\mathrm{NN})}(r) = \frac{f_{\pi\mathrm{NN}}^2}{4\pi} \left(\frac{m_\pi}{2M_N} \right)^2 (2\mathbf{I}^2 - 3) \cdot \frac{2\mathbf{S}^2 - 3}{3} \frac{e^{-m_\pi r}}{r} \quad . \tag{10}$$

For the deuteron channel ($I = 0$, $S = 1$) this reduces to

$$V_\pi(r)^{(\mathrm{NN})} = -\frac{g_{\pi\mathrm{NN}}^2}{4\pi} \frac{e^{-m_\pi r}}{r} \quad , \tag{11}$$

with

$$g_{\pi\mathrm{NN}}^2 \equiv f_{\pi\mathrm{NN}}^2 \left(\frac{m_\pi}{2M_N} \right)^2 \simeq 0.073 \quad . \tag{12}$$

Observe that $g_{\pi\mathrm{NN}}$ is tiny relative to $g_{\sigma\mathrm{NN}}$ and $g_{\omega\mathrm{NN}}$.

For the $\rho(770)$ interaction the lagrangian is

$$\mathcal{L}_{\rho\mathrm{NN}} = g_{\rho\mathrm{NN}} \bar{N} \gamma_\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu N \quad . \tag{13}$$

where we take $g_\rho^2/4\pi = 2.0$ [18]. The $\rho(770)$ component of the potential energy is isospin dependent,

$$V_\rho^{(\mathrm{NN})}(r) = \frac{g_{\rho\mathrm{NN}}^2}{4\pi} (2\mathbf{I}^2 - 3) \frac{e^{-m_\rho r}}{r} \quad . \tag{14}$$

1. **The S=1, I=0 (Deuteron) Channel**

Of the four component potentials, only the $\omega(783)$ piece is repulsive in the $I = 0$ channel. It is mainly the interplay of the large $\omega(783)$ repulsion and $\sigma(600)$ attraction that results in the deuteron binding. Although the coefficient of the $\omega(783)$ potential is larger than that of the attractive $\sigma(600)$, the lighter mass of the $\sigma(600)$ allows it to dominate over $\omega(783)$ at intermediate distance scales. The $\omega(783)$ provides the repulsive core. The potential energy which includes all four components is displayed in Fig. 1. When we use as input potential energies $V_\sigma^{(NN)}$, $V_\omega^{(NN)}$, $V_\rho^{(NN)}$ and $V_\pi^{(NN)}$ to fit the value of $g_{\sigma NN}^2/4\pi$ to the deuteron binding energy, the value obtained for $g_{\sigma NN}$ is

$$\frac{g_{\sigma NN}^2}{4\pi} \simeq 10.83 \quad . \tag{15}$$
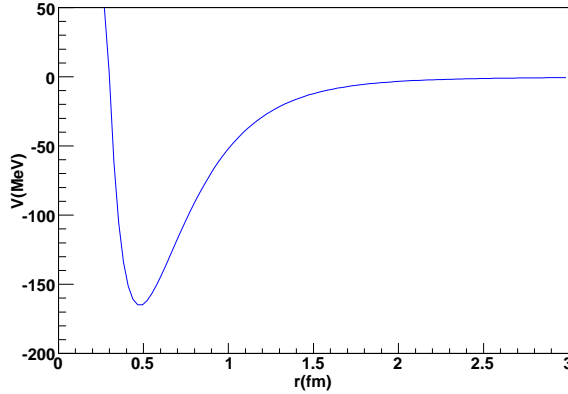
This is in accord with the range given above in Eq. (7).

FIG. 1: Deuteron Potential Energy.

2. **The S=0, I=1 Channel**

The two-nucleon system has isospins $I = 0, 1$ and spins $S = 0, 1$, and if the two nucleons are in an S-wave, the $I = S = 0$ and $I = S = 1$ configurations are ruled out by Fermi-Dirac statistics. The deuteron has $I = 0$, $S = 1$, which leaves finally the combination $I = 1, S = 0$. The $I = 1, S = 0$ channel has a virtual bound state, as evidenced by the large scattering length $a(^1S_0) \simeq 23.7$ fm. Since $\sigma(600)$ and $\omega(783)$ are isoscalars, a potential energy with only these exchanges would imply equal binding in both the $I = 0, S = 1$ and $I = 1, S = 0$ channels. However, the $\rho(770)$ contribution is attractive in the deuteron channel but repulsive (at one third the strength) in the $I = 1, S = 0$

channel. Our numerical study obtains the desired result of no binding for the $I = 1, S = 0$ channel upon using the $\sigma(600)$ coupling strength of Eq. (15).

A. Anthropic Implications

Anthropics can be viewed as relating to the class of theories (including possibly string theory and chaotic inflation) in which spacetime has a domain structure, each domain with its own set of fundamental parameters (CKM mixing angles, lepton and quark masses, *etc*) [19]. Such a description is often referred to as the *Multiverse* [20, 21]. This theoretical possibility provides an interesting framework for viewing deep issues of physics, such as the cosmological constant [22], the Higgs vacuum expectation value [23] and the light quark masses [1].

Previous studies have revealed that even a modest variation of fundamental parameters can induce qualitative changes in physical systems [1, 23]. Works like these can help determine the 'physical' windows in parameter space which enable Universes like our own to exist. This is in contrast with 'theoretical' parameter windows which can be associated with a given underlying theory, *e.g.* string dynamics. If the physical window is small compared to the theoretical window, then it is not unreasonable to assign a flat probability distribution across the physical window and thus to study the relative liklihood of that our Universe should exist within a given theoretical framework [20].

In this paper, we are concerned with the light quark masses. Refs. [1, 16] show that increasing $m_u + m_d$ decreases the σNN coupling whereas decreasing $m_u + m_d$ has the opposite effect. For example, it is estimated that taking $m_\pi^2 \to 0$ produces a 40% increase in $g_{\sigma NN}^2/4\pi$ while increasing m_π^2 to twice its physical value decreases $g_{\sigma NN}^2/4\pi$ by 20%. The source of this sensitivity to m_π (and ultimately to $m_u + m_d$) lies mainly in the threshold contribution in the dispersive integrals of Eqs. (2),(3). By contrast, the ρNN and ωNN couplings are relatively unaffected by such changes in the light quark masses because their spectral functions are large only well above threshold.

1. *Unbinding of the Deuteron via Weakening of $V_\sigma^{(NN)}$*: In our description, the two-nucleon potential is mainly the competition between the attractive V_σ and the repulsive V_ω. It is not hard to upset this balance and thus unbind the deuteron by varying some combination of the potential energies, *e.g.* by reducing $V_\sigma^{(NN)}$ (keeping all other interactions fixed), increasing $V_\omega^{(NN)}$ (again keeping all other interactions fixed), *etc*. However, the former is the only variation associated with the mass values of the light quarks. To help keep track of how the deuteron binding energy responds to the variation in $g_{\sigma NN}$, we find it useful to define the

$g^2_{\sigma NN}/(4\pi)$	$\mathrm{BE}_{\mathrm{deut}}$ (MeV)	$R^{(\mathrm{coup})}$	$R^{(\mathrm{BE})}$
10.83	2.226	1.	1.
10.81	2.032	0.998	0.913
10.71	1.422	0.989	0.639
10.63	1.019	0.982	0.457
10.59	0.813	0.977	0.365
10.53	0.611	0.972	0.274
10.47	0.408	0.967	0.183
10.38	0.206	0.959	0.092
10.32	0.105	0.953	0.047
10.21	0.012	0.943	0.006

TABLE I: Dependence of deuteron binding energy on the σNN coupling as expressed directly (first two columns) or in terms of the ratios $R^{(\mathrm{coup})}$, $R^{(\mathrm{BE})}$ defined in Eq. (16).

following ratios,

$$R^{(\mathrm{coup})}[g_{\sigma NN}] \equiv \frac{g^2_{\sigma NN}/(4\pi)}{10.83} \qquad R^{(\mathrm{BE})}[BE_{\mathrm{deut}}] \equiv \frac{\mathrm{BE}_{\mathrm{deut}}\ (\mathrm{MeV})}{2.263} \quad . \tag{16}$$

From the numerical values in Table I, we see that the deuteron becomes unbound if $V_\sigma^{(\mathrm{NN})}$ is reduced in strength by about 6%. This is to be compared with the 10% required to disassociate heavy nuclei [1].

2. *Alternative Variations*: There can, in principle, be other parameter variations. For the sake of completeness, we briefly consider a few of these.

It is obvious that increasing $V_\sigma^{(\mathrm{NN})}$ will increase the deuteron binding energy. However, there is another consequence, which turns out to somewhat alter the physical world. We find that an increase of about 8% in $V_\sigma^{(\mathrm{NN})}$, with other potentials held fixed, suffices to first produce binding the $I = 1, S = 0$ NN channel.

Varying the ω and ρ potentials would likewise have physical consequences, *e.g.* with $V_\sigma^{(\mathrm{NN})}$ fixed, an increase in strength of $V_\omega^{(\mathrm{NN})}$ by about 7% would result in unbinding the deuteron, This kind of variation is not associated in any obvious way with the fundamental parameters of the Standard Model and thus has no connections with our anthropic analysis.
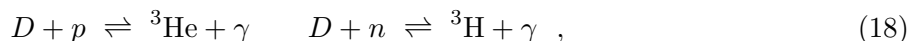
IV. CONCLUSION

Our aim in this paper has been to explore the dependence of deuteron binding on variations in $m_u + m_d$ in terms of a simple model involving just central potentials. The central potentials were constructed with care, by exploiting up-to-date results from studies of nuclear binding. This description is admittedly in contrast with the traditional approach which uses the one-pion-exchange (OPE) tensor potential [7]. We feel that these two procedures are not incompatible. Consider a description containing the huge σ and ω potentials along with the tensor OPE. If the σ and ω effects cancel even more than in our calculation (which would involve a slight change in V_σ), the deuteron binding would be given largely in terms of the tensor OPE. Even so, deuteron binding would still feel the changes induced by $m_u + m_d$ largely via the effect on V_σ (as described in this paper). We leave for future study a more ambitious description which includes all these elements in a Schrödinger equation context, plus insights from chiral perturbation theory.

Thus, given the deuteron potential energies of Eq. (5), we have shown (*cf* Table I) that a slight reduction in V_σ results in unbinding the deuteron. Even a slight weakening of the attractive component results in a large effect. As a result, the deuteron is a sensitive system for studying how physics would respond to changes in $m_u + m_d$. The crucial links in the chain of logic which connects these are (i) the chiral relation, $m_\pi^2 = B_0(m_u + m_d)$, which relates the pion mass to the light quark masses and (ii) the dispersive formula Eq. (2) which relates V_σ to the pion mass [10]. The 6% decrease in V_σ needed to unbind deuterons is less than the estimated 10% needed to unbind heavy nuclei [1].
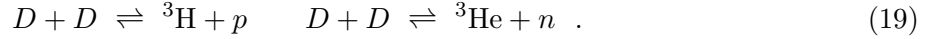
It is abundantly clear that if conditions forbade atoms or nuclei from existing, then there would be no Universe as we know it. The absence of the deuteron, although itself just a solitary quantum particle, would likewise be dramatic. The deuteron first appears in Cosmology as part of Big Bang Nucleosynthesis (BBN) in the early Universe,

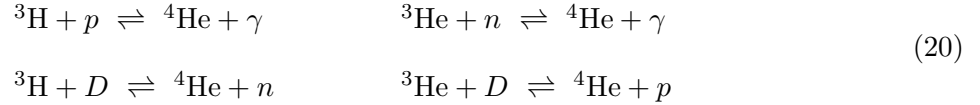$$p + n \; \rightleftharpoons \; D + \gamma \; . \tag{17}$$

Before deuteron formation, free neutrons are copious (about one neutron per five protons), but after formation free neutrons are absent (the number of free neutrons equals the number of deuterons at about 200 seconds after the Big Bang). Once deuterons are formed, they take part in a number of additional BBN reactions, such as fusion with protons and neutrons,

$$D + p \; \rightleftharpoons \; {}^3\text{He} + \gamma \qquad D + n \; \rightleftharpoons \; {}^3\text{H} + \gamma \; , \tag{18}$$

or with each other,

$$D + D \;\rightleftharpoons\; {}^3\text{H} + p \qquad D + D \;\rightleftharpoons\; {}^3\text{He} + n \;\; . \tag{19}$$

These lead to the production of ^{4}He,

$$
\begin{aligned}
{}^3\text{H} + p \;&\rightleftharpoons\; {}^4\text{He} + \gamma & {}^3\text{He} + n \;&\rightleftharpoons\; {}^4\text{He} + \gamma \\
{}^3\text{H} + D \;&\rightleftharpoons\; {}^4\text{He} + n & {}^3\text{He} + D \;&\rightleftharpoons\; {}^4\text{He} + p
\end{aligned}
\tag{20}
$$

Although trace amounts of heavier nuclei are formed, the above relations display the basic yield of BBN once the deuteron is formed. It is clear that, although just a 'solitary' particle, the deuteron occupies a central role in BBN.[2]

There are some additional issues, not yet mentioned, which we leave for further study. As regards BBN, it would be interesting to study more carefully what happens under conditions where the deuteron becomes slightly unbound. In a different direction, by using a modification of the meson exchange approach described in this paper, we can consider the possibility of molecular structures in the meson sector. Of particular interest are the states $a_0(980)$ and $f_0(980)$, whose nearness to the $K\bar{K}$ threshold has suggested a molecular interpretation [24]. We will report on this study elsewhere [25].

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